The quantum mechanics of particle-correlation measurements in high-energy heavy-ion collisions

Alejandro Ayala,[a] Gordon Baym,[b] and James L. Popp[b,c]

[a] *Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, Aptdo. Postal 70-543, México D.F. 04510*

[b] *Department of Physics, University of Illinois at Urbana-Champaign, 1110 W. Green St., Urbana, IL 61801*

[c] *Department of Physics, New York University, 4 Washington Place, New York, NY 10003*

(February 9, 2008)

The Hanbury Brown–Twiss (HBT) effect in two-particle correlations is a fundamental wave phenomenon that occurs at the sensitive elements of detectors; it is one of the few processes in elementary particle detection that depends on the wave mechanics of the produced particles. We analyze here, within a quantum mechanical framework for computing correlations among high-energy particles, how particle detectors produce the HBT effect. We focus on the role played by the wave functions of particles created in collisions and the sensitivity of the HBT effect to the arrival times of pairs at the detectors, and show that the two detector elements give an enhanced signal when the single-particle wave functions of the detected particles overlap at both elements within the characteristic atomic transition time of the elements. The measured pair correlation function is reduced when the delay in arrival times between pairs at the detectors is of order of or larger than the transition time.

I. INTRODUCTION

The Hanbury Brown–Twiss (HBT) effect in nuclear and particle physics [1–6] is the enhancement at small relative momentum in the probability for observing pairs of identical bosons drawn from the same collision event, compared to that for pairs chosen from different events. Underlying the correlations between the fluctuations at two nearby but separate detectors is the symmetry of the wave function of a system of identical bosons under interchange of any two particles. HBT particle interferometry is in a special class of high energy physics experiments that, like kaon regeneration and neutrino oscillation searches, study an effect basic to quantum mechanics: amplitude interference. However, while, e.g., kaon regeneration experiments examine how weak-interactions affect the *internal* degrees of freedom of kaons, HBT interferometry probes the many-body *spatial* wave function of the final state particles. The effect, as we emphasize here, is fundamentally a *wave phenomenon* manifested *at the detectors*. Indeed, as is well understood in terms of classical wave mechanics, HBT intensity-intensity correlations may be observed with any type of wave satisfying the superposition principle (e.g., sound waves) with at least two independent incoherent sources and two detectors.

In this paper we analyze from a quantum mechanical point of view the physics of correlations between identical particles, and how the detectors in high-energy physics experiments probe the many-particle wave functions in space and time. While the spatial requirements on the detector separations for measuring an HBT effect are well understood, the temporal requirements are less well studied. We may ask, what is the maximum separation in time of arrival of two identical particles at the detectors that will still yield an HBT effect between the two particles? It is commonly assumed in collision experiments that interference happens only between particles produced in the same event, not between particles from different events. One could imagine that this occurs because the correlations are built in at the time of the event; however since one detects interferometry between photons from the opposite edges of a star, where there can clearly be no correlation between the emission processes, this is certainly not the case. Underlying the assumption of lack of interferometry between particles from different events is the existence of a timescale for detection of interferometry, which is exceeded by successive events. As we show here, the crucial timescale for HBT correlations is that inherent in the detection process; the maximum value of the measured HBT pair correlation function is reduced when pairs of waves arrive at the detector atoms with delay times comparable to or larger than the characteristic transition times of the atoms in the detector.

Considering the wave mechanics of the emitted particles in space and time also enables us to understand more fully questions such as: How does the exchange symmetry of the many-particle wave function lead to detection of momentum correlations? How does the wave packet nature of the single-particle states affect measurements of the HBT effect? For example, how does the spreading of wave packets affect the measurement of correlations?

1

Part of the motivation to examine the role of the interplay of the time scales of the pion field and the detection process in particle interferometry experiments is the clear illustration in the original studies of photon correlations of how detection times affect the size of the maximum value of the measured two-photon correlation function [7,8]. As Purcell [9] pointed out, for chaotic stationary sources the maximum height of this function above unity is approximately the ratio of the coherence time of the light to the observational binning time. Hanbury Brown and Twiss's measurements on the star Sirius [10] produced a maximum correlation signal $\sim 10^{-6}$, in contrast to data on like-pion pair correlations gathered to date, with final-state interaction corrections included, which indicate that the two-pion correlation function, $C_2(|\vec{q}|)$, rises approximately (but not unambiguously) to two in the lowest bin in relative momentum.

Four time scales are basic in a measurement of the HBT effect. The first is the *lifetime* of the source, $\tau_{\rm s}$, over which it emits particles; in a high-energy collision the production region exists for $\sim$ 1-10 fm/c [$\sim 0.3 - 3 \times 10^{-23}$ sec] in its own rest frame. Second is the *coherence time* of the source, $\tau_{\rm c}$, the characteristic time for each elementary radiator to emit a wave. Generally, the formation time for a pion or other particle depends on its energy; the characteristic lifetime of elementary pion sources $\tau_{\rm c}$ is $\lesssim 1$ fm/c, and $\tau_{\rm c} \lesssim \tau_{\rm s}$. The third scale is the *atomic transition time*, $\tau_{\rm atomic}$, of the energy-absorbing material of the detector, the time over which a mobile electric charge is created in the detector; on this time scale the detector atoms "do quantum mechanics" on the incoming particle waves, i.e., the atoms are sensitive to the amplitudes and phases of the waves. For ionization of a gas atom, $\tau_{\rm atomic} \sim \hbar/10\,{\rm eV} \sim 10^{-16}$ sec. The final scale is the *exposure time*, $\tau_{\rm exp}$, the interval between observations of the state of the detector atoms. The time of an accurate momentum measurement on a relativistic charged particle in a magnetic spectrometer of length $\ell_{\rm spec}$ is $\sim \ell_{\rm spec}/c$, which for a typical length scale of $10\,{\rm m}$ gives a characteristic time to measure the momentum $\tau_{\rm exp} \sim 3 \times 10^{-8}$ sec. A more familiar but less relevant measurement time scale is the detector *resolution time*, $\tau_{\rm res}$, literally, the minimum time to detect an electronic signal, which includes the time it takes to collect and amplify the initial electric charge. A typical resolution time for a wire chamber, essentially the rise time of the voltage pulse produced by an electron avalanche on an anode wire, is on a nanosecond scale, $\tau_{\rm res} \sim 10^{-9}$ sec. As we show in Sect. III, $\tau_{\rm atomic}$, rather than $\tau_{\rm res}$, is the important time scale over which HBT correlations are detected.

This paper is organized as follows: In Section II, we develop a framework for understanding the temporal structure in correlations between quantum mechanical particles created in high-energy experiments. To be specific, we describe pions, but our results hold for other particles as well. In Section III, we analyze the features of relativistic wave packets important to HBT, and compute the detection probability for pairs directly-produced in a heavy-ion collision and then for those from direct production plus resonance decay. In Section IV, we summarize our results and conclusions. In the Appendices, we describe the details of the quantum mechanics of particle detection.

II. DESCRIPTION OF THE PROBLEM

One and two pion measurements on a multiparticle system are described by the single-pion and two-pion density matrices for particles of given charge: $\langle \phi^\dagger(x_1)\phi(x_2) \rangle$ and $\langle \phi^\dagger(x_1)\phi^\dagger(x_2)\phi(x_3)\phi(x_4) \rangle$, where $\phi(x)$ is the part of the (Heisenberg representation) pion field operator that destroys particles of the given charge, and the brackets indicate an ensemble average over the states of the colliding nuclei. The pion-pair correlation function, $C_2(\vec{q})$, which depends directly on these two functions, is measured as the ratio of the pion pair distribution to the separate single distributions:

$$C_2(\vec{q}) = \frac{\{d^6 n_2/dp^3 dp'^3\}}{\{d^3 n_1/dp^3 \ d^3 n_1/dp'^3\}}, \tag{1}$$

where $\vec{q} = (\vec{p} - \vec{p}')/2$. The braces in the numerator denote an average over an ensemble of pairs drawn from the same event and in the denominator they denote an average over an ensemble of pairs drawn from different events. The single-pion momentum distribution $d^3 n_1/dp^3$ is given in terms of the plane-wave momentum state creation and annihilation operators $a_{\vec{p}}^\dagger$ and $a_{\vec{p}}$ by

$$\frac{d^3 n_1}{dp^3} = \langle a_{\vec{p}}^\dagger a_{\vec{p}} \rangle = 2\varepsilon_p \int d^3 r_1 \, d^3 r_2 \ e^{-ip\cdot(x_1-x_2)} \langle \phi^\dagger(x_1)\phi(x_2) \rangle, \tag{2}$$

with $\varepsilon_p = (\vec{p}^2 + m^2)^{1/2}$ and $p \cdot x \equiv \omega t - \vec{p} \cdot \vec{r}$, where ω is the particle energy and $\vec{p}$ is the momentum. Note that the explicit dependence on t_1 and t_2 in the phase factor of Eq. (2) is canceled by the time-dependence of the single-pion density matrix, as later shown in Eq. (10). The momentum distribution of pairs is given similarly by

$$\frac{d^6 n_2}{dp^3 dp'^3} = \langle a_{\vec{p}}^\dagger a_{\vec{p}'}^\dagger, a_{\vec{p}'}, a_{\vec{p}} \rangle = 4\varepsilon_p \varepsilon_{p'} \int d^3 r_1 \cdots d^3 r_4 \; e^{-ip\cdot(x_1 - x_4)} e^{-ip'\cdot(x_2 - x_3)} \; \langle \phi^\dagger(x_1)\phi^\dagger(x_2)\phi(x_3)\phi(x_4)\rangle. \tag{3}$$

A detailed description of the pion wave functions that emerge from the production region of a heavy-ion collision requires knowledge of the evolution and geometry of the source and how field excitations of quarks, gluons, nucleons, etc. form the currents that radiate pions. However, these details are unimportant for our present purpose of understanding how we detect HBT. We retain only the essential features of the pion production process, in particular its characteristic length and time scales. In high-energy collisions, pions are produced in bremsstrahlung-like processes, in states similar to momentum wave packet states. Final-state interactions, particularly Coulomb and strong interactions, severely influence particle state evolution; also, collisions between pions and air molecules, detector materials, and other target nuclei can significantly alter C_2; in particular, multiple scatterings of particles after they leave the source redistribute pion tracks thereby reducing the size of the measured correlation in each bin in relative momentum and decreasing the effective size of the source measured in HBT experiments [11,12]. Thus in order to focus on the wave mechanics of detecting HBT, we assume here that after their last strong interaction the wave packets propagate in vacuum.

The production of charged pions and their propagation to the detectors is described by the Klein-Gordon equation for the charged pseudoscalar field, which relates the pion field, ϕ, to the source, $J(x)$, of the pion field at the *last* strong interaction:

$$\left(\partial^2/\partial t^2 - \nabla^2 + m_\pi^2\right)\phi(x) = -J(x), \tag{4}$$

where $J^\dagger$ is the part of the current operator that emits pions of the given charge. Thus

$$\phi(x) = \int dx' \; D_{\text{ret}}(x - x')J(x'), \tag{5}$$

where the free-field retarded Green's function, $D_{\text{ret}}(x - x')$, satisfies

$$\left(\partial^2/\partial t^2 - \nabla^2 + m_\pi^2\right)D_{\text{ret}}(x - x') = -\delta^{(4)}(x - x'), \tag{6}$$

and vanishes for $t < t'$; the integrations are over all space and time. The Green's function has the representation

$$D_{\text{ret}}(x - x') = \int \frac{d^4 k}{(2\pi)^4} \frac{e^{-ik\cdot(x - x')}}{(\omega + i\epsilon)^2 - \varepsilon_k^2}, \tag{7}$$

where ϵ is a positive infinitesimal and $\omega = k^0$. Integrating over ω for $t > t'$, we find that the created pion field is given by

$$\phi(x) = \int \frac{d^3 k}{2i\varepsilon_k (2\pi)^3} \int_{-\infty}^{t} dt' \int d^3 r' \; J(x')\, e^{-ik\cdot(x - x')}, \tag{8}$$

with k on-shell. Since one measures at t much larger than the source lifetime we extend the upper limit in the time integral in Eq. (8) to $+\infty$ and write

$$\phi(x) = \int \frac{d^3 k}{2i\varepsilon_k (2\pi)^3} \; e^{-ik\cdot x} \; J(k), \tag{9}$$

where $J(k)$ is the Fourier transform in space and time of the pion source current operator. Equations (2) and (3) imply that the Lorentz-invariant one pion momentum distribution is given by

$$2\varepsilon_p \frac{d^3 n_1}{dp^3} = \int dx_1 dx_2 \; e^{-ip\cdot(x_1 - x_2)} \langle J^\dagger(x_1)J(x_2)\rangle \tag{10}$$

and the two pion momentum distribution by

$$4\varepsilon_p \varepsilon_{p'} \frac{d^6 n_2}{dp^3 dp'^3} = \int dx_1 dx_2 dx_3 dx_4 \; e^{-ip\cdot(x_1 - x_4) - ip'\cdot(x_2 - x_3)} \langle J^\dagger(x_1)J^\dagger(x_2)J(x_3)J(x_4)\rangle; \tag{11}$$

Eqs. (10) and (11) show how the HBT correlation function directly probes the correlation functions of the source, a point of view initially introduced in Ref. [6].

3

The HBT effect arises only when single particles are produced in *mixed* quantum states, that is, in an ensemble or statistical mixture of single-particle states. (See Ref. [13] for further discussion of the multiparticle states in HBT.) When particles are produced *completely independently*, e.g., by a thermal source, the source currents factorize as

$$\langle J^\dagger(x_1)J^\dagger(x_2)J(x_3)J(x_4)\rangle = \langle J^\dagger(x_1)J(x_4)\rangle\langle J^\dagger(x_2)J(x_3)\rangle + \langle J^\dagger(x_1)J(x_3)\rangle\langle J^\dagger(x_2)J(x_4)\rangle. \tag{12}$$

The pion correlation functions similarly factorize, and the momentum distribution of pion pairs [15] is

$$\frac{d^6 n_2}{dp^3 dp'^3} = \frac{d^3 n_1}{dp^3}\frac{d^3 n_1}{dp'^3} + \frac{1}{4\varepsilon_{\vec{p}}\varepsilon_{\vec{p}'}}\left|\langle J^\dagger(p)J(p')\rangle\right|^2; \tag{13}$$

HBT interferometry seeks to measure the second term in Eq. (12). The key physical mechanism that leads to the factorization of currents is the loss of phase correlations among the elementary sources, which is expected to occur in heavy-ion collisions through the considerable rescattering of pions in the production region.[1] The HBT effect is maximum, for incoherent emission.

By contrast, when particles are produced *completely coherently*, e.g., as in an atom laser beam extracted from a Bose-Einstein condensate [16] or by an ideal chiral condensate [17], the source currents factorize as

$$\langle J^\dagger(x_1)J^\dagger(x_2)J(x_3)J(x_4)\rangle = \langle J^\dagger(x_1)\rangle\langle J^\dagger(x_2)\rangle\langle J(x_3)\rangle\langle J(x_4)\rangle. \tag{14}$$

In this case, single particles are produced in a *pure* quantum state; the momentum distribution of pion pairs is the product of two single-pion momentum distributions, and the HBT effect would be absent.

III. MEASURING CORRELATION FUNCTIONS

We turn now to the physics of momentum measurements. How ionization chambers (and many other detectors used in high-energy physics) track well-localized particles is understood from semi-classical ideas; however, the response of such detectors to a particle whose wave function is not well-localized in space and time necessarily requires a quantum mechanical description (a problem of longstanding interest, e.g., from Ref. [18] to [19].) HBT correlations are the direct result of the interactions of the *many-pion* wave function with the electrons of the energy absorbing material of the detector. Computing the probability of measuring the momentum of a fast charged particle is a problem in multiple scattering, where we must compute the probability of measuring a particle track through a spectrometer system. The first interaction selects out the direction of the momentum, and subsequent interactions with more atoms and a magnetic field select out the magnitude of the momentum. We define the *single-pion momentum measurement probability* $P_\pi^{\rm a}(\vec{k})$ as the probability for a pion to ionize a detector gas atom at some location $\vec{a}$, say the first atom along a track, and undergo a transition to a plane-wave momentum state $\vec{k}$. Similarly, we define the *pion-pair momentum measurement probability* $P_{\pi\pi}^{\rm ab}(\vec{k},\vec{k}')$ as the probability for each pion to ionize one atom and emerge in a plane wave state, one atom at $\vec{a}$ with a pion in the final state $\vec{k}$ and one atom at $\vec{b}$ with a pion in the final state $\vec{k}'$.

As derived in Appendix A, the crucial function describing the response [20] of the detector in measuring a particle of momentum $\vec{k}$ – a "momentum measurement" as defined above – is the *spectrometer function*,

$$\mathcal{S}_{\vec{k}}^{\rm a}(x_1\, x_2) \equiv e^4\,\psi_{\vec{k}}(x_1)\,\langle\tilde{j}_{\rm a}^\dagger(x_1)\,\tilde{j}_{\rm a}(x_2)\rangle\,\psi_{\vec{k}}^*(x_2), \tag{15}$$

where $\tilde{j}_{\rm a}(x)$ is the effective electromagnetic current operator for the atomic electrons, Eq. (51), and $\psi_{\vec{k}}$ is the final pion plane-wave momentum state wave function. As shown by detailed calculation in Appendices A and B, the momentum measurement probabilities are given by the pion correlation function as filtered by the "spectrometer function," i.e., the overlap in space and time of the single and two-pion correlation functions (for given final pion states) with the correlation function of the effective electron currents in the detector atoms,

$$P_\pi^{\rm a}(\vec{k}) = \int dx_1 dx_2\,\mathcal{S}_{\vec{k}}^{\rm a}(x_1\, x_2)\,\langle\phi^\dagger(x_1)\phi(x_2)\rangle, \tag{16}$$

---

[1]In the absence of rescattering the statistics of the constituents of the currents can destroy this factorization; a simple example is the correlations among pions radiated by a weakly-interacting gas of nucleons [11,14]; see also [13].

and

$$P_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') = \int dx_1 dx_2 dx_3 dx_4\; \mathcal{S}_{\vec{k}}^{\mathrm{a}}(x_2\,x_3)\; \langle\phi^\dagger(x_1)\phi^\dagger(x_2)\phi(x_3)\phi(x_4)\rangle\; \mathcal{S}_{\vec{k}'}^{\mathrm{b}}(x_1\,x_4)\,. \tag{17}$$

When the source term factorizes as in Eq. (12), we obtain the pair momentum detection probability as a sum of two terms,

$$P_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') = D_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') + E_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}')\,, \tag{18}$$

where the *direct* term is

$$D_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') = \int dx_1 dx_2 dx_3 dx_4\; \mathcal{S}_{\vec{k}}^{\mathrm{a}}(x_2\,x_3)\,\mathcal{S}_{\vec{k}'}^{\mathrm{b}}(x_1\,x_4)\; \langle\phi^\dagger(x_2)\phi(x_3)\rangle\langle\phi^\dagger(x_1)\phi(x_4)\rangle, \tag{19}$$

and the *exchange* term is given by

$$E_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') = \int dx_1 dx_2 dx_3 dx_4\; \mathcal{S}_{\vec{k}}^{\mathrm{a}}(x_2\,x_3)\,\mathcal{S}_{\vec{k}'}^{\mathrm{b}}(x_1\,x_4)\; \langle\phi^\dagger(x_1)\phi(x_3)\rangle\langle\phi^\dagger(x_2)\phi(x_4)\rangle\,. \tag{20}$$

The direct term is the product of the single-pion detection probabilities, Eq. (16),

$$D_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') = P_\pi^{\mathrm{a}}(\vec{k})P_\pi^{\mathrm{b}}(\vec{k}')\,, \tag{21}$$

and is equivalent to the probability of observing two like-charge pions from two different collision events, one in state $\vec{k}$ at $\vec{a}$ and one in state $\vec{k}'$ at $\vec{b}$. The exchange term is the HBT effect.

Knowledge of the mixture of pion states produced by the source is critical to understanding HBT interferometry. For pions produced in single particle states $\varphi_i(x)$ the single-pion correlation function is

$$\langle\phi^\dagger(x_1)\phi(x_2)\rangle = \sum_i F_i\;\varphi_i^*(x_1)\,\varphi_i(x_2)\,, \tag{22}$$

where F_i specifies how the distribution of pion states depends on the evolving geometry of the particle production region. [Generally, the decomposition of the single pion correlation function as a sum of single particle states defines the single particle states.] The corresponding pion source current-current correlation function is

$$\langle J^\dagger(x_1)J(x_2)\rangle = \sum_i F_i\;\mathcal{J}_i^*(x_1)\,\mathcal{J}_i(x_2)\,, \tag{23}$$

where $\mathcal{J}_i(x)$ is the transition matrix element of the pion source operator $J(x)$. To illustrate the structure concretely, we approximate the sources of the pions produced in a high-energy collision as Gaussians in space and time

$$\mathcal{J}_i(x) = \frac{N}{(2\pi)^2\tau_{\mathrm{c}}R_{\mathrm{c}}^3}e^{-ip\cdot(x-x_0)}e^{-(t-t_0)^2/2\tau_{\mathrm{c}}^2}e^{-(\vec{r}-\vec{r}_0)^2/2R_{\mathrm{c}}^2}, \tag{24}$$

where R_{c} and τ_{c} are the characteristic length and time scales for pion formation, and N is a normalization constant; here the subscript i stands for the central momentum $\vec{p}$ and the space-time origin of the wave x_0. It has been shown elsewhere [4,11,21] that the effective relative momentum scale of the pair correlation function involves not only the dimensions and lifetime of the particle production region and the mixture of states produced but also the individual particle formation length and time scales. These sources give rise to pions in Gaussian wave packets, which in the far field have the form

$$\varphi_i(x) \approx -\frac{1}{8\pi^2 r}\int_{m_\pi}^\infty d\varepsilon_q\;\mathcal{J}_i(\varepsilon_q,q\hat{r})\,e^{-i\varepsilon_q t+iqr}\,, \tag{25}$$

where $\mathcal{J}_i(\varepsilon_q,q\hat{r})$ is the four-dimensional Fourier transform of $\mathcal{J}_i(x)$. Equation (25) follows from Eq. (9) by selecting the outgoing wave after integrating over momentum directions.

For $\vec{p}=p\hat{z}$, the Gaussian source gives rise to a wave packet which spreads out approximately as

$$\langle\vec{r}_\perp^2\rangle = R_{\mathrm{c}}^2 + \frac{t^2}{(R_{\mathrm{c}}\varepsilon_p)^2} \tag{26}$$

$$\langle(z-v_p t)^2\rangle = \frac{1}{2}\left(R_{\mathrm{c}}^2 + v_p^2\tau_{\mathrm{c}}^2\right) + \frac{t^2}{2\varepsilon_p^2\left(R_{\mathrm{c}}^2 + v_p^2\tau_{\mathrm{c}}^2\right)\gamma_p^4}\,, \tag{27}$$

5

where $\gamma_p = \varepsilon_p/m_\pi$ and $v_p = |\vec{p}|/\varepsilon_p$. For $t \gg \tau_{\rm c}$, the ratio of the longitudinal to the transverse spread of the wave function is $\sim 1/\gamma_p^2$. Far from the source, relativistic pion momentum wave packets are pancake-shaped and move in a direction perpendicular to the face of the pancake. Consider, e.g., a $1\,{\rm GeV}$ pion and assume that $\tau_{\rm c} \sim R_{\rm c} \sim 1\,{\rm fm}$. For a source-detector distance $L \sim 1\,{\rm m}$ the size of the wave packet envelope at the detectors in the transverse direction is $\sim 20\,{\rm cm}$ and is $\sim 0.2\,{\rm cm}$ thick in the longitudinal direction.

Substituting Eq. (22) into Eqs. (19) and (20), we find the direct and exchange terms,

$$D_{\pi\pi}^{\rm ab}(\vec{k}\,\vec{k}\,') = \sum_{ij} F_i F_j \, \mathcal{W}_{\vec{k}}^{\rm a}(ii)^* \, \mathcal{W}_{\vec{k}'}^{\rm b}(jj) \tag{28}$$

$$E_{\pi\pi}^{\rm ab}(\vec{k}\,\vec{k}\,') = \sum_{ij} F_i F_j \, \mathcal{W}_{\vec{k}}^{\rm a}(ij)^* \, \mathcal{W}_{\vec{k}'}^{\rm b}(ij) \, , \tag{29}$$

where we define the wave function-detector overlap function

$$\mathcal{W}_{\vec{k}}^{\rm a}(ij) \equiv \int dx dx' \, \varphi_i^*(x) \, \mathcal{S}_{\vec{k}}^{\rm a}(x\,x') \, \varphi_j(x') \, . \tag{30}$$

Each possible pair of pion waves (ij) contributes to the direct term, with one detected at $\vec{a}$ and the other at $\vec{b}$. However, only pairs of waves that arrive together at each detector atom, within the time interval set by the detector functions $\mathcal{S}$ contribute to the exchange term. Interferometry occurs when the wave functions of each of the particles overlap in each of the detectors at the same times. Equation (29), with (28), shows that there is no restriction on the time interval between the arrivals at the two atoms. Correlations are measured over the time scale imposed by the response of the atoms, Eq. (30), the time scale over which the atoms are sensitive to the *phase* and *amplitude* of the incoming particle waves.

We illustrate the physics of detecting correlations with a simplified model of the effective atomic current-current correlation function in Eq. (15), which takes into account the main features of the dynamic response of atoms to ionization by relativistic charged particles [21], namely, we assume that the space and time dependence of the effective atomic current-current correlation function factorizes as

$$\langle \widetilde{j}_{\rm a}^\dagger(x_1) \, \widetilde{j}_{\rm a}(x_2) \rangle = f_{\rm a}(\vec{r}_1\,\vec{r}_2\,) \, g(t_1\,t_2) \, . \tag{31}$$

Since the statistical distribution of atomic electron states in a gas under normal conditions is essentially time-independent, then $g(t_1\,t_2) = g(t_1 - t_2)$. The Fourier transform of $g(t)$ is the atomic energy-absorption spectrum, and is determined from the distribution for energy-loss per ionization for a relativistic charged particle passing through a gas. In a monatomic gas, this distribution is approximately Gaussian with a long tail extending up to the kinematic limit for energy transfer, $2m_e(\beta\gamma)^2$. The majority of ionizing collisions occur within the Gaussian part of the distribution and it is particularly the ionization events in which the free electron carries away a minimal amount of kinetic energy that are important for tracking. Thus, we can neglect the high-energy tail and select a (normalized) Gaussian energy spectrum for g:

$$g(t) = \int \frac{d\omega}{2\pi} \, \tilde{g}(\omega) \, e^{-i\omega t}, \tag{32}$$

where $\tilde{g}(\omega) = \sqrt{2\pi} \, \zeta^{-1} \, e^{-(\omega - \omega_0)^2/2\zeta^2}$. The average energy loss, ω_0, for creating electron-ion pairs in, e.g., noble gases, is $10 - 50\,{\rm eV}$ [22]. The energy bandwidth of the Gaussian part of the distribution, ζ, is $\sim 10\,{\rm eV}$, an energy on the scale of the average ionization potential per electron in a Thomas-Fermi model of an atom. This scale determines the characteristic time for ionization, $\tau_{\rm atomic} = 1/\zeta \sim 10^{-16}\,{\rm sec}$. The spatial scale of the atomic correlation function is determined by the sizes of the atomic electron wave functions, characteristically $\sim 1\,{\rm \AA}$; in terms of momentum, if the kinetic energy picked up by an electron is a few tens of eV then the electron momentum, $|\vec{q}|$, is on a keV$/c$ scale, corresponding to a distance $1/|\vec{q}| \sim 1\,{\rm \AA}$, a result roughly consistent with the size of impact parameters required in a classical Weizsäcker-Williams picture of a collision. For convenience, we model $f_{\rm a}(\vec{r}_1\,\vec{r}_2\,)$ as the product of two Gaussian functions centered on the atomic nucleus:

$$f_{\rm a}(\vec{r}_1\,\vec{r}_2\,) \equiv f_{\rm a} \, e^{-(\vec{r}_1 - \vec{a}\,)^2/2R_{\rm a}^2} \, e^{-(\vec{r}_2 - \vec{a}\,)^2/2R_{\rm a}^2} \, , \tag{33}$$

where $R_{\rm a}$ is of order angstroms and $f_{\rm a}$ is a constant.

6

The delays between the emissions of pions directly produced in a high-energy nucleus-nucleus collision are no larger than the lifetime of the source. However those produced in resonances can have a longer spread in emission times. As we now show, Hanbury Brown-Twiss correlations are insensitive to the delays in arrival times at the detector atoms generated by emission delays much smaller than the atomic response time, τ_{atomic}. With the use of the far-field form of the wave functions for directly produced pions, Eq. (25), with Eq. (24), the space and time integrations in Eq. (30) are trivial. Integrating over the energy components of both pion wave packets we find the intermediate form,

$$
\mathcal{W}_{\vec{k}}^{\text{a}}(ij) = c_a \int \frac{d\omega}{2\pi}\, \tilde{g}(\omega)\, e^{-i(\varepsilon_k + \omega)(t_0 - t_0')}
$$
$$
\times\, e^{-(\vec{k} - q\hat{a})^2 R_{\text{a}}^2}\, e^{iq\hat{a}\cdot(\vec{r}_0 - \vec{r}_0')} \mathcal{J}_{\vec{p}}^*(\varepsilon_k + \omega\,, q\hat{a})\, \mathcal{J}_{\vec{p}'}(\varepsilon_k + \omega\,, q\hat{a})\,, \tag{34}
$$

with $c_a = \pi e^4 f_{\text{a}} R_{\text{a}}^6 / 4a^2 \varepsilon_k V$; $|\vec{q}|$ is determined by the condition $\varepsilon_q = \varepsilon_k + \omega$, and $t_0 - t_0'$ is the emission delay time between the pion waves. Since as one sees from the singles distribution [23], the transition matrix elements vary over an MeV scale or more, and $\tilde{g}(\omega)$ restricts ω to a neighborhood of size ζ about ω_0, both on an eV scale, then for pions of energy at least one GeV, it is a very good approximation to replace q with k and neglect ω everywhere in the second line of Eq. (34). Consequently, the first term on the second line of Eq. (34) requires that $\hat{k} \cdot \hat{a} \approx 1$ (to within one part in at least 10^{10}), so that $\vec{k} = k\hat{a}$. The integral over ω is then simply the Fourier transform of $\tilde{g}$:

$$
g(t_0 - t_0') = e^{-i\omega_0(t_0 - t_0')} e^{-(t_0 - t_0')^2 \zeta^2 / 2}\,, \tag{35}
$$

and the overlap function is

$$
\mathcal{W}_{\vec{k}}^{\text{a}}(ij) \approx c_a g(t_0 - t_0')\, \mathcal{J}_{\vec{p}x_0}^*(\varepsilon_k\,, \vec{k})\, \mathcal{J}_{\vec{p}'x_0'}(\varepsilon_k\,, \vec{k})\,. \tag{36}
$$

The direct term is independent of g, since $t_0 = t_0'$ and $\vec{p} = \vec{p}'$ in Eq. (36). In fact, Eqs. (35) and (36) show that when the emission delay time between pions is much less than $1/\zeta = \tau_{\text{atomic}}$ we may also neglect the time dependence of the detection process in the exchange term; to a very good approximation we may then write $|g(t_0 - t_{0'})| \sim |g(0)| = 1$. Substituting Eq. (36) into Eqs. (28) and (29) and summing over the mixture of wave packet states we find that the pair momentum detection probability is proportional to the Fourier transform of Eq. (12),

$$
P_{\pi\pi}^{\text{ab}}(\vec{k}\,, \vec{k}') = c_a c_b \left[\langle J^\dagger(k) J(k)\rangle \langle J^\dagger(k') J(k')\rangle + |\langle J^\dagger(k) J(k')\rangle|^2\right]\,, \tag{37}
$$

the naive result obtained by neglecting the time dependence of the detection process. However, in situations leading to much longer time delays, in particular, pion emission from long-lived resonances compared with direct pion production, one cannot necessarily neglect the time dependence of the detection process.[2]

Consider interferometry between a π^- produced directly in a heavy-ion collision and from the decay of, say a lambda, $\Lambda \to \pi^- + p$, produced in the same reaction. Because the Λ moves more slowly than a directly produced pion of the same rapidity as the one emitted in the decay, the pion from decay will lag the directly produced one by a time δ at the detector atoms. To estimate this arrival time delay, we note that a π^- emitted in the forward direction has rapidity $y_0 \approx 0.67$ in the Λ rest frame, and that a Λ of rapidity y travels on average a distance $\tau_\Lambda \sinh y$ before decaying, where τ_Λ is the Λ lifetime. Thus, $\delta = \tau_\Lambda / (\cosh y + \sinh y / \tanh y_0)$, which for a Λ of typical rapidity 3 is $\sim 0.037\tau_\Lambda = 9.7 \times 10^{-12}$ sec, much longer than the atomic time scale. Pions emitted in other than the forward direction will have an even greater time lag. As we shall see, detector atoms are sensitive to such delays when we measure pair correlations.

The reduction in the probability for detecting pairs due to time delays between direct and resonance decay pions can be readily estimated using a simplified scalar-field model to compute the overlap, Eq. (30), with a detector atom of the wave functions from a direct-production pion and a pion from, say, lambda decay. The wave mechanical features of the exact problem do not depend on the details of the model. Consider the interaction $\mathcal{H}_I(x) = \alpha\, \pi^\dagger(x)\bar{p}(x)\Lambda(x)$, where α is the coupling constant. The transition matrix element for the decay-product pion is

$$
\mathcal{J}_\pi^\Lambda(x) = \alpha\, \psi_p^*(x)\varphi_\Lambda(x)\,, \tag{38}
$$

---

[2] Effects of time delays in propagation of wave packets can also enter in the observation of neutrino oscillations, as noted by Kim [24], and references therein.

where φ_Λ is the wave function of the Λ baryon of width $\Gamma \sim 10^{-6}\,\mathrm{eV}$, and ψ_p is a plane wave state for the proton with momentum $\vec{p}$. As before, we write the wave functions of the lambda and both pions in the far-field form, Eq. (25). The Fourier transform of Eq. (38) is

$$\mathcal{J}_\pi^\Lambda(\varepsilon\,,\vec{q}) = \frac{\alpha}{2\varepsilon_{\vec{q}+\vec{p}}^\Lambda \sqrt{2\varepsilon_{\vec{p}}^p V}} \frac{\mathcal{J}_\Lambda(\varepsilon + \varepsilon_{\vec{p}}^p, \vec{q} + \vec{p})}{(\varepsilon + \varepsilon_{\vec{p}}^p - \varepsilon_{\vec{q}+\vec{p}}^\Lambda) + i\Gamma/2}\,, \tag{39}$$

where $\mathcal{J}_\Lambda$, the source function for lambda production, is similar in structure to Eq. (24), $\varepsilon_{\vec{q}}^\Lambda = (\vec{q}^2 + m_\Lambda^2)^{1/2}$, and $\varepsilon_{\vec{p}}^p = (\vec{p}^2 + m_p^2)^{1/2}$. We subsume the origin and momentum labels of the wave packets into the subscript on the transition matrix element, $\mathcal{J}_\Lambda$. Thus, the wave function overlap at a detector atom located at $\vec{a}$ with pion final state $\vec{k}$ is

$$\mathcal{W}_{\vec{k}}^{\mathrm{a}}(\pi\,\Lambda\,\vec{p}) \approx c_a \frac{\alpha\,\mathcal{J}_\pi^*(\varepsilon_{\vec{k}}^\pi, \vec{k})\mathcal{J}_\Lambda(\varepsilon_{\vec{k}}^\pi + \varepsilon_{\vec{p}}^p, \vec{k} + \vec{p})}{2\varepsilon_{\vec{k}+\vec{p}}^\Lambda \sqrt{2\varepsilon_{\vec{p}}^p V}} \int \frac{d\omega}{2\pi} \frac{\tilde{g}(\omega)}{(\omega + \varepsilon_{\vec{k}}^\pi + \varepsilon_{\vec{p}}^p - \varepsilon_{\vec{k}+\vec{p}}^\Lambda) + i\Gamma/2}\,. \tag{40}$$

The overlap function $\mathcal{W}_{\vec{k}'}^{\mathrm{b}}(\pi\,\Lambda\,\vec{p})$ for an atom at $\vec{b}$ is exactly the same as above, but with $\vec{k}'$ in place of $\vec{k}$.

The contribution to the exchange term, given by

$$P_{\pi\,\pi(\Lambda)}^{\mathrm{exch}}(\vec{k}\,,\vec{k}') = \sum_{\pi\,\Lambda\,\vec{p}} F_\pi F_\Lambda\,\mathcal{W}_{\vec{k}}^{\mathrm{a}}(\pi\,\Lambda\,\vec{p})^* \,\mathcal{W}_{\vec{k}'}^{\mathrm{b}}(\pi\,\Lambda\,\vec{p}), \tag{41}$$

decreases as the width Γ of the lambda decreases, i.e., as the lifetime of the lambda increases. We can see this explicitly by selecting a convenient form for the energy absorption spectrum of the atom:

$$\tilde{g}(\omega) = \frac{\zeta}{(\omega - \omega_0)^2 + (\zeta/2)^2}\,, \tag{42}$$

where $\zeta \sim 10\,\mathrm{eV}$ (cf. Eq. (32)). The effect is simplest to see for $\vec{k} = \vec{k}'$ (we report the calculation for $\vec{k} \neq \vec{k}'$ in a later paper); then the sum in Eq. (41) is

$$\sim \sum_{\pi\,\Lambda} F_\pi F_\Lambda \int \frac{d^3p}{(2\pi)^3 2\varepsilon_{\vec{p}}^p} \frac{|\mathcal{J}_\pi^*(\varepsilon_{\vec{k}}^\pi, \vec{k})\mathcal{J}_\Lambda(\varepsilon_{\vec{k}}^\pi + \varepsilon_{\vec{p}}^p, \vec{k} + \vec{p})|^2}{(2\varepsilon_{\vec{k}+\vec{p}}^\Lambda)^2} \frac{1}{(\omega_0 + \varepsilon_{\vec{k}}^\pi + \varepsilon_{\vec{p}}^p - \varepsilon_{\vec{k}+\vec{p}}^\Lambda)^2 + (\zeta + \Gamma)^2/4}\,. \tag{43}$$

Since the factor containing Γ varies over a much smaller energy scale than that of the rest of the integrand we can replace the last term in (43) with $2\pi\delta(\varepsilon_{\vec{k}}^\pi + \varepsilon_{\vec{p}}^p - \varepsilon_{\vec{k}+\vec{p}}^\Lambda)/(\zeta + \Gamma)$, where we have also neglected ω_0. It is convenient to assume that the Λ is produced in a spherically symmetric state in the lab, so that the Fourier transform of $\mathcal{J}_\Lambda$ only depends on energy. The width of the decaying particle to lowest-order in our model interaction is $\Gamma = \alpha^2 p_0/8\pi m_\Lambda^2$, where $p_0 = ((m_x^2/2m_\Lambda)^2 - m_p^2)^{1/2}$, and $m_x^2 = m_\Lambda^2 + m_p^2 - m_\pi^2$. Thus, integrating over proton momentum we find

$$P_{\pi\,\pi(\Lambda)}^{\mathrm{exch}}(\vec{k}\,,\vec{k}) = \frac{\Gamma}{\Gamma + \zeta}\mathcal{P}_{\pi\,\pi(\Lambda)}^{\mathrm{exch}}(\vec{k}\,,\vec{k}) \tag{44}$$

where

$$\mathcal{P}_{\pi\,\pi(\Lambda)}^{\mathrm{exch}}(\vec{k}\,,\vec{k}) \equiv c_a c_b \frac{m_\Lambda^2}{2p_0 k} \sum_{\pi\,\Lambda} F_\pi F_\Lambda\,|\mathcal{J}_\pi^*(\varepsilon_{\vec{k}}^\pi, \vec{k})\mathcal{J}_\Lambda(\varepsilon_{q_0}^\Lambda\,, q_0)|^2 \ln\Big|\frac{\varepsilon_{q_0}^\Lambda + \beta_\pi q_0}{\varepsilon_{q_0}^\Lambda - \beta_\pi q_0}\Big|, \tag{45}$$

with $q_0 = p_0 m_\Lambda/m_\pi$, $\beta_\pi \gamma_\pi = |\vec{k}|/m_\pi$, and replacing $\mathcal{J}_\Lambda$ with an average matrix element. Equation (45) is the probability obtained when the atomic response time is neglected, that is, when one takes $\tilde{g}(\omega) = 2\pi\delta(\omega - \omega_{\mathrm{o}})$. Equation (44) shows that pions from the decay of long-lived resonances can lead to arrival time delays at the detector atoms, relative to direct-production pions, that do indeed give a reduced contribution to the HBT signal. Thus, when $\Gamma \ll \zeta$ the $\pi\Lambda$ exchange term is reduced by a factor Γ/ζ (for lambda-decay pions, $\sim 10^{-7}$) from the naive result and for short-lived resonances with $\Gamma \gg \zeta$ we recover the case where we neglect the detector response time. Among the common weak decays that produce pions the detection time effect is strongest for interferometry with charged π's from mesons with $\Gamma < 10^{-7}\mathrm{eV}$; for $K^\pm$, K_L^0, $\Gamma/\zeta < 10^{-8}$. We expect a weaker suppression for pairs from the

8

shorter-lived particles where $\Gamma < 10^{-5}$eV; for K_S^0, Λ, and $\Sigma^\pm$, $\Gamma/\zeta < 10^{-6}$. For rarer decays like $\Xi^\pm$, Ω^-, and heavier flavors with $\Gamma \lesssim 10^{-3}$eV the detection time suppression factors are less than 10^{-4}.

The full exchange term, of which Eq. (44) is a part, includes all possible pion pairs which involve contributions of the type: $(\mathcal{J}_\pi^*, \mathcal{J}_{\pi'})$, $(\mathcal{J}_\pi^*, \mathcal{J}_{\Lambda'})$, $(\mathcal{J}_\Lambda^*, \mathcal{J}_{\pi'})$, and $(\mathcal{J}_\Lambda^*, \mathcal{J}_{\Lambda'})$. The $(\mathcal{J}_\pi^*, \mathcal{J}_{\pi'})$ term is computed in Eq.(36). The terms $(\mathcal{J}_\pi^*, \mathcal{J}_{\Lambda'})$ and $(\mathcal{J}_\Lambda^*, \mathcal{J}_{\pi'})$ are identical and contribute a factor 2 times Eq. (44) to the full exchange term. The $(\mathcal{J}_\Lambda^*, \mathcal{J}_{\Lambda'})$ term may be computed in a way similar to that of Eq. (44), giving an even smaller contribution to the HBT effect. The essential result is that the response time of the detector atoms is sensitive to phase differences between the waves due to arrival time delays at the detectors; a result independent of the origin of the waves. One can see this, quite simply, by introducing an explicit time delay in the wave funtions at the detectors in Eq.(30).

IV. SUMMARY AND CONCLUSIONS

We have developed, within an elementary quantum mechanical framework for computing correlation measurements in high-energy experiments, a general description of how detectors probe many-particle wave functions. The HBT effect is the consequence of wave mechanics performed by particle detectors and depends only on the wave functions of the particles at the sensitive elements of the detectors; it is not caused by stimulated emission or any other mechanism at the source and it does not depend on the history of the particles, e.g., the particles do not have to have a common origin.

We have studied how momentum correlations between pairs of particles are detected via the HBT effect. We have shown that: 1) The like-pair correlation function is able to reveal momentum correlations because the single-particle wave functions of the detected particles overlap at at least two sensitive elements of a detector, within the characteristic atomic transition times of those elements. 2) There is no restriction on the time interval between the transitions of the two detector atoms. 3) The size of the measured pair correlation function is reduced when the delay in arrival times between pairs at the detectors is of order of or larger than the transition time; e.g., delays from particles produced in very long-lived resonance decays.

ACKNOWLEDGEMENTS

APPENDIX A: MOMENTUM DETECTION PROBABILITY FOR SINGLE CHARGED PIONS

We compute here the probability for an energetic charged pion to ionize a gas atom and emerge in a final plane-wave state with momentum $\vec{k}$. We denote the initial state by $|\mathcal{I}\rangle = |I, i_a, 0\rangle$, where I is the inital pion state, and $|i_a\rangle$ is the initial state of the atomic electrons; and the final state by $|\mathcal{F}\rangle = |\vec{k}, f_a\rangle$ where $|f_a\rangle$ contains the same number of electrons as $|i_a\rangle$, but with one electron in a continuum state. For simplicity, we work with single-pion states; the generalization to many-pion states is straightforward. The interaction of a charged pion with a detector-gas atom located at a space point $\vec{a}$ is

$$\mathcal{H}_I = \mathcal{H}_I^\pi + \mathcal{H}_I^a, \tag{46}$$

where $\mathcal{H}_I^\pi = ej_\pi^\mu A_\mu$, A_μ is the electromagnetic field, j_π^μ is the charged pion electromagnetic current, $\mathcal{H}_I^a = ej_a^\mu A_\mu$, with j_a^μ the electron current of the detector atom and $-e$ the electron charge; we work in the interaction picture. For $\pi^\pm$, the electromagnetic current is $j_\pi^\mu(x) = \pm\phi^\dagger(x)i\overleftrightarrow{\partial}^\mu\phi(x)$.

The lowest-order contribution to the amplitude for ionization in a collision between an energetic charged pion and an atom comes from the second order terms in the matrix element of the time evolution operator, $U(t, t')$,

$$\mathcal{A}_\pi^a(\mathcal{I} \to \mathcal{F}) = -\int dx_1 dx_2 \langle\mathcal{F}|T\left[\mathcal{H}_I^a(x_1)\mathcal{H}_I^\pi(x_2)\right]|\mathcal{I}\rangle = -ie^2 \int dx_1 dx_2 \langle f_a|j_a^\mu(x_1)|i_a\rangle D_{\mu\nu}(x_1\,x_2)\langle\vec{k}|j_\pi^\nu(x_2)|I\rangle. \tag{47}$$

Here T denotes time-ordering; the time integrations are from t to t', where the exposure time $\tau_{\exp}$ is $t' - t$, and the space integrals are over all space. In the latter expression we use the free-field forms for the operators, and introduce the photon propagator, $D_{\mu\nu}(x_1\,x_2) = -i\langle 0|T\left[A_\mu(x_1)A_\nu(x_2)\right]|0\rangle$, where $|0\rangle$ is the vacuum.

The transition matrix element of the pion current becomes $\langle \vec{k}|\phi^\dagger(x_2)i\overleftrightarrow{\partial}_2^\nu\phi(x_2)|I\rangle = \psi_{\vec{k}}^*(x_2)i\overleftrightarrow{\partial}_2^\nu\langle 0|\phi(x_2)|I\rangle$, where $\psi_{\vec{k}}(x) = (1/\sqrt{2\varepsilon_k V})e^{-ik\cdot x}$. Since the energy spectrum of the atomic states important for tracking via ionization only has Fourier components a few tens of eV or less, the spectrum of the atomic function $\int dx_1\langle f_a|j_a^\mu(x_1)|i_a\rangle D_{\mu\nu}(x_1,x_2)$ is similarly constrained. Integrating in x_2 by parts in Eq. (47) and neglecting the derivatives of this function, we can replace $i\overleftrightarrow{\partial}_2^\nu$ by $2k^\nu$.

Squaring the transition amplitude, summing over all final electron states, averaging over initial pion and electron states, we find the ionization probability

$$P_\pi^a(\vec{k}) = 4e^4 \int dx_1 dx_2 dx_3 dx_4 D_{\alpha\sigma}^*(x_3\,x_4)\langle j_a^\alpha(x_3)j_a^\mu(x_1)\rangle D_{\mu\nu}(x_1\,x_2)\psi_{\vec{k}}^*(x_2)\psi_{\vec{k}}(x_4)k^\sigma k^\nu\langle\phi^\dagger(x_4)\phi(x_2)\rangle, \qquad (48)$$

where the single-pion density matrix is

$$\langle\phi^\dagger(x_4)\phi(x_2)\rangle = \sum_I \rho_I\langle I|\phi^\dagger(x_4)\phi(x_2)|I\rangle, \qquad (49)$$

with ρ_I the probability that the state $|I\rangle$ is produced by the source; the electric current-electric current correlation function for the atomic electrons is

$$\langle j_a^\alpha(x_3)j_a^\mu(x_1)\rangle \equiv \sum_i \rho_i^a \sum_f \langle i_a|j_a^\alpha(x_3)|f_a\rangle\langle f_a|j_a^\mu(x_1)|i_a\rangle, \qquad (50)$$

where ρ_i^a is the probability of finding the atom at $\vec{a}$ in the state $|i_a\rangle$. Defining an effective current operator,

$$\widetilde{j}_a(x_2) \equiv 2k^\nu \int dx_1 j_a^\mu(x_1)D_{\mu\nu}(x_1\,x_2), \qquad (51)$$

we see that the ionization probability reduces to

$$P_\pi^a(\vec{k}) = \int dx_1 dx_2 \mathcal{S}_{\vec{k}}^a(x_1\,x_2)\langle\phi^\dagger(x_1)\phi(x_2)\rangle, \qquad (52)$$

where the spectrometer function $\mathcal{S}_{\vec{k}}^a(x_1\,x_2)$ is defined in Eq. (15).

APPENDIX B: MOMENTUM DETECTION PROBABILITY FOR CHARGED PION PAIRS

In this Appendix we compute the probability for detecting a pair of π^+: one with momentum $\vec{k}$ at $\vec{a}$ and one with momentum $\vec{k}'$ at $\vec{b}$. We denote the initial and final states by $|\mathcal{I}\rangle = |I,i_a,i_b,0\rangle$ and $|\mathcal{F}\rangle = |\vec{k},\vec{k}',f_a,f_b,0\rangle$, where the number of pions in the initial and final states are the same, and $\vec{k}$ and $\vec{k}'$ are the measured states. The interaction Hamiltonian for the system of detector atom-a and detector atom-b plus two pions is $\mathcal{H}_I = \mathcal{H}_I^\pi + \mathcal{H}_I^a + \mathcal{H}_I^b$, where the interactions are given in Appendix A. Two-pion correlation measurements are given by the term fourth-order in $\mathcal{H}_I$ in the time evolution operator, $U(t,t')$, which with free fields becomes

$$\frac{e^4}{2}\int dx_1 dx_2 dx_3 dx_4\, j_a^\mu(x_1)T\left[j_\pi^\nu(x_2)j_\pi^\sigma(x_3)\right]j_b^\kappa(x_4)T\left[A_\mu(x_1)A_\nu(x_2)A_\sigma(x_3)A_\kappa(x_4)\right]. \qquad (53)$$

The matrix elements of the pion and photon operators are reduced as follows: We write the pion current-current matrix element as

$$\langle\vec{k},\vec{k}'|\,j_\pi^\nu(x_2)j_\pi^\sigma(x_3)|I\rangle = i\overleftrightarrow{\partial}_2^\nu i\overleftrightarrow{\partial}_3^\sigma\langle\vec{k},\vec{k}'|\phi^\dagger(x_2)\phi(x_2)\phi^\dagger(x_3)\phi(x_3)|I\rangle, \qquad (54)$$

where $\overleftrightarrow{\partial}_2^\nu$ and $\overleftrightarrow{\partial}_3^\sigma$ act between the operators carrying the same variables. Using the free-field commutation relation $\left[\phi(x_2),\phi^\dagger(x_3)\right] = \mathcal{D}(x_2\,x_3)$, a c-number, we express the pion-current operators in terms of the two-pion correlation operator plus a term involving $\mathcal{D}(x_2\,x_3)$. The matrix element

$$\langle\vec{k},\vec{k}'|\phi^\dagger(x_2)\phi^\dagger(x_3)\phi(x_3)\phi(x_2)|I\rangle \qquad (55)$$

10

is exactly the interaction of two pions ionizing two atoms; we remove two pions at two separate space-time points and replace them. The commutator term $\mathcal{D}(x_2\,x_3)$ does not contribute to lowest-order scattering, hence we replace the matrix element on the right-hand-side of Eq. (54) with (55); whereupon we eliminate the time-ordering of the pion fields in Eq. (53) using the symmetry of these expressions under the interchange $x_2 \leftrightarrow x_3$ and $\nu \leftrightarrow \sigma$. Since we require $\psi_{\vec{k}}$ at $\vec{a}$ and $\psi_{\vec{k}'}$ at $\vec{b}$, (55) finally becomes $\psi_{\vec{k}}^*(x_2)\psi_{\vec{k}'}^*(x_3)\langle 0|\phi(x_3)\phi(x_2)|I\rangle$. The vacuum expectation value of the time-ordered photon operators in Eq. (53) factorizes as

$$- \langle 0|T\left[A_\mu(x_1)A_\nu(x_2)A_\sigma(x_3)A_\kappa(x_4)\right]|0\rangle$$
$$= D_{\mu\nu}(x_1,x_2)D_{\kappa\sigma}(x_4,x_3) + D_{\mu\sigma}(x_1,x_3)D_{\kappa\nu}(x_4,x_2) + D_{\mu\kappa}(x_1,x_4)D_{\nu\sigma}(x_2,x_3). \tag{56}$$

The last term in Eq. (56) represents photon exchange between detector atoms and is not important for ionization.

After eliminating the crossed photon lines in the amplitude, and the derivatives of the pion fields as in Appendix A, we write

$$\mathcal{A}_{\pi\pi}^{\mathrm{ab}}(\mathcal{I} \to \mathcal{F}) = -e^4 \int dx_1 dx_2 dx_3 dx_4 \langle f_{\mathrm{a}}|\,j_{\mathrm{a}}^\mu(x_1)|i_{\mathrm{a}}\rangle D_{\mu\nu}(x_1\,x_2)\langle f_{\mathrm{b}}|\,j_{\mathrm{b}}^\kappa(x_4)|i_{\mathrm{b}}\rangle D_{\kappa\sigma}(x_4\,x_3)$$
$$\times \psi_{\vec{k}}^*(x_2)\psi_{\vec{k}'}^*(x_3)4k^\nu k'^\sigma\langle 0|\phi(x_3)\phi(x_2)|I\rangle. \tag{57}$$

The transition probability is computed in exactly the same way as for single pion detection; redefining the electron currents, Eq. (51), and expressing the momentum measurements in terms of spectrometer functions, Eq. (15), we derive:

$$P_{\pi\pi}^{\mathrm{ab}}(\vec{k},\vec{k}') = \int dx_1 dx_2 dx_3 dx_4 \mathcal{S}_{\vec{k}}^{\mathrm{a}}(x_2\,x_3)\langle \phi^\dagger(x_1)\phi^\dagger(x_2)\phi(x_3)\phi(x_4)\rangle \mathcal{S}_{\vec{k}'}^{\mathrm{b}}(x_1\,x_4). \tag{58}$$

[1] G. Goldhaber, S. Goldhaber, W. Lee, and A. Pais, Phys. Rev. 120 (1960) 300.
[2] B. Andersson et al., Physica Scripta 34 (1986) 451.
[3] D. H. Boal, C. H. Gelke, and B. K. Jennings, Rev. Mod. Phys. 62 (1990) 553.
[4] U. A. Wiedemann and U. Heinz, submitted to Phys. Rep., nucl-th/9901094.
[5] U. Heinz and B. V. Jacak, Ann. Rev. Nucl. Part. Sci. 49 (1999).
[6] M. Gyulassy, S. K. Kauffmann, and L. W. Wilson, Phys. Rev. C20 (1979) 2267.
[7] R. Hanbury Brown and R. Q. Twiss, Nature 177 (1956) 27; Nature 178 (1956) 1447; Nature 179 (1957) 1128.
[8] R. Hanbury Brown, The Intensity Interferometer: its Application to Astronomy (Taylor & Francis LTD, London, 1974).
[9] E. M. Purcell, Nature 178 (1956) 1449.
[10] R. Hanbury Brown and R. Q. Twiss, Nature 178 (1956) 1046.
[11] G. Baym, Acta Phys. Pol. B29 (1998) 1839.
[12] A. Ayala, G. Baym, B. V. Jacak, J. L. Popp, and B. Vanderheyden, to be published.
[13] A. Makhlin and E. Surdutovich, Phys. Rev. C59 (1999) 2761.
[14] A. Ayala, G. Baym, and J. L. Popp, to be published.
[15] S. Pratt, Phys. Rev. Lett. 53 (1984) 1219; Phys. Rev. D33 (1986) 72; Phys. Rev. D33 (1986) 1314; Phys. Rev. C42 (1990) 2646.
[16] M.-O. Mewes et al., Phys. Rev. Lett. 78 (1997) 582; M. R. Andrews et al., Science 275 (1997) 637; H.-J. Miesner et al., Science 279 (1998) 1005.
[17] J. D. Bjorken, K. Kowalsky, and C. C. Taylor, SLAC-PUB-6109 (1993); K. Rajagopal and F. Wilczek, Nucl. Phys. B379 (1993) 395; S. Gavin, Nucl. Phys. A590 (1995) 163c; J. D. Bjorken, Acta Phys. Pol. B28 (1997) 2773.
[18] N. F. Mott, Proc. Roy. Soc. London, Ser. A 126 (1929) 79.
[19] A. Makhlin and Yu. Sinyukov, Sov. J. Nucl. Phys. 46 (1987) 354.
[20] R. J. Glauber, Quantum Optics and Electronics, Les Houches 1964 Summer School, ed. C. DeWitt, A. Blandin and C. Cohen-Tannoudji (Gordon and Breach, New York, 1965).
[21] J. L. Popp, Ph.D. Dissertation, University of Illinois, 1997 (unpublished).
[22] T. Ferbel, Experimental Techniques in High-Energy Nuclear and Particle Physics (World Scientific, Singapore, 1991).
[23] Y. Akiba et al., Nucl. Phys. A610 (1996) 139c.
[24] C. W. Kim, J. Korean Phys. Soc. 29 (1996) s157.